

THURSDAY, JANUARY 1, 1885

THE "AMERICAN JOURNAL OF MATHEMATICS"

American Journal of Mathematics, Pure and Applied.
Published under the Auspices of the Johns Hopkins University. Vols. v., vi., vii., Part I. (Baltimore : Isaac Friedenwald, 1882-4.)

THE general features of this *Journal* have been clearly indicated in the notices of the previous volumes (see NATURE, vol. xxii. p. 73, vol. xxvii. p. 193), and we need only remark under this head that these original characteristics have been maintained throughout the numbers now under our consideration.

Prof. Sylvester was the editor-in-chief until his return to this country ; now the mantle has fallen upon his successor, Prof. Newcomb, under whose auspices vol. vii. is being published. Dr. Thomas Craig has been the assistant editor during the issue of all the numbers.

The chief papers treat of the higher algebra. In this branch the contributions of Prof. Sylvester naturally loom large. They are "On Sub-Invariants, i.e. Semi-Invariants to Binary Quantics of an Unlimited Order," "Tables of Generating Functions, reduced and representative for certain Ternary Systems of Binary Forms" (the "Tables" were calculated by Messrs. Durfee and Ely), "A Constructive Theory of Partitions, arranged in Three Acts, an Interact, and an Exodion," a most valuable contribution to the theory, written with the author's characteristic fervour, but perhaps the gem of the collection is the first instalment of the "Lectures on the Principles of Universal Algebra."

We naturally turn next to the papers by Prof. Cayley. These are a "Note on a Partition-Series," "A Memoir on Seminvariants," following up a "remarkable" discovery by Capt. Macmahon, which leads to the conclusion that the theory of seminvariants is a part of that of symmetric functions, and three sets of tables, viz. non-unitary partition tables, seminvariant tables, and tables of the symmetric functions of the roots, to the degree 10 for the form—

$$1 + bx + cx^2/1 \cdot 2 + \dots = (1 - ax)(1 - \beta x)(1 - \gamma x) \dots$$

Following in the wake of these Leviathans, Mr. Durfee contributes "Tables of the Symmetric Functions of the Twelfthic," and "The Tabulation of Symmetric Functions"; Capt. Macmahon writes on "Seminvariants and Symmetric Functions," "Symmetric Functions of the 13^{ic}," and "On Perpetuants"; he is also the author of a short "Note on the Development of an Algebraic Fraction," the moving cause of which is a previous article by M. Faà de Bruno, entitled "Sur le développement des fonctions rationnelles," which in its turn owed its origin to a note by Prof. Sylvester in the *Johns Hopkins Circulars*. Mr. J. Hammond, another worker in this field, has a paper "On the Solution of the Differential Equation of Sources," in which he gives a disproof of Prof. Sylvester's fundamental postulate, a discovery which he first communicated to the London Mathematical Society. Mr. G. S. Ely applies the method of graphs to compound partitions, and Mr. Morgan Jenkins gives a proof of a theorem in partitions, and furnishes a note on Prof.

VOL. XXXI.—NO. 792

Sylvester's constructive theory of partitions, mentioned above.

We pass from this group of subjects, which centres more especially round the name of Sylvester, and come to papers on elliptic functions in one or other of the forms under which that branch is now ranged. M. Faà de Bruno has a long article on "Quelques applications de la théorie des formes binaires aux fonctions elliptiques"; Dr. Craig contributes several papers, viz. "Some Elliptic Function Formulae," "On a Theta-Function Formula," "On Quadruple Theta-Functions" (two papers), "On Theta-Functions with Complex Characteristics," and "On Certain Groups of Relations satisfied by the Quadruple Theta-Functions." Prof. W. W. Johnson presents a proof of the imaginary period in elliptic functions; Mr. A. L. Daniels communicates three notes on Weierstrass's methods in the theory of these functions; and Prof. Cayley, in a memoir on the abelian and theta functions, reproduces, with additional developments, the course of lectures which he delivered at the Johns Hopkins University in the early months of 1882.

The other papers on algebraical subjects may be grouped together. They are:—"On Division of Series," by Rev. J. Hagen; "Tables for Facilitating the Determination of Empirical Formulae," by A. W. Hale; "On the Development of an Algebraic Fraction," by Dr. Franklin; some papers "On the Theory of Numbers," by A. S. Hathaway; "Sur une formule relative à la théorie des fonctions d'une variable," by M. Hermite; "Calculus of Direction and Position," by E. W. Hyde; "Compound Determinants," by C. A. Van Velzer (written before the author had seen Mr. R. F. Scott's paper in vol. xiv. of the London Mathematical Society's *Proceedings*), in which is discussed Picquet's proof of a theorem of Sylvester's. Mr. McClintock writes on the resolutions of equations of the fifth degree, a subject which is also handled by Mr. G. P. Young, who in addition discusses the principles of the solution of equations of the higher degrees. Mr. G. S. Ely furnishes some notes on the numbers of Bernoulli and Euler (adopting a name given by Sylvester), and gives a useful bibliography of Bernoulli's numbers. Such lists as these are of great service to workers.

Dr. Story defines the absolute classification of loci to be that classification which is not altered by any real linear transformation, and which is identical with the ordinary classification in so far as the latter is independent of all consideration of the nature of the infinite elements of the loci; a part of this classification has been made (as Dr. Story remarks) in essence by Prof. Sylvester in the *Phil. Mag.* (February 1851). The title of the paper is "On the Absolute Classification of Quadratic Loci, and on their Intersections with each other and with Linear Loci." The same author also contributes two articles on the non-Euclidian geometry: one is a continuation of a paper by him in vol. iv., and in it are given a number of formulae relating to distances, angles, areas, and volumes; the other is entitled "Non-Euclidian Properties of Conics," and contains an application of Prof. Cayley's projective measurement, generalised by Klein, and still further extended by the author in the paper just cited, to the conic.

Dr. Franklin discusses some points in the theory of

K

cubic curves by a novel method, but not many new theorems are the result ; and Mr. E. W. Davis gives an expression for the co-ordinates of a point on a binodal quartic curve as rational functions of the elliptic functions of a variable parameter.

The only purely geometrical article is one by Mr. B. Alvord, entitled "The Intersection of Circles and the Intersection of Spheres." The problems discussed are to draw a circle which shall make a given angle with three given circles ; to draw a sphere which shall cut each of four given spheres at a given angle ; and then two Steinerian problems, viz. to draw a circle which shall cut four given circles at the same angle (angle unknown), and the analogous problem for five spheres. The number of solutions in each case is given, and there are four plates containing thirteen figures. Prof. C. H. Smith supplies a graphic method of solving spherical triangles.

There is a single astronomical article on certain possible abbreviations in the computation of the long-period inequalities of the moon's motion due to the direct action of the planets, by Mr. G. W. Hill, who states that Hansen has characterised the calculation of the coefficients of these inequalities as extremely difficult, but he himself thinks that, if the shortest methods are followed, there is no ground for such an assertion.

Prof. Turazza gives a note (which the editor had mislaid for three years), "Di un nuovo teorema relativo alla rotazione di un corpo ad un asse."

The only physical paper is Prof. Rowland's, "On the Propagation of an Arbitrary Electro-magnetic Disturbance on Spherical Waves of Light and the Dynamical Theory of Diffraction." The classical paper by Stokes "On the Dynamical Theory of Diffraction" is discussed ; in addition the author treats of the general problem of spherical waves of light, which he has not seen considered anywhere else.

We think the titles of the papers and a perusal of their contents quite bear out Mr. Glaisher's opinion, pronounced in his notice of the previous volumes (vol. xxvi. *ubi supra*), viz. that "the volumes represent a considerable amount of mathematical work, a fair proportion of which may have real influence on the advancement of the science." Some readers might like to have a more diversified bill of fare set before them, but no one can say that what is offered is not generally first class. The form of the *Journal* lends itself admirably to the important tables with which it has been enriched from its earliest days. We are glad to find this young work maintaining its early promise, and we wish for it even a higher success in the days to come.

A SYSTEM OF PSYCHOLOGY

A System of Psychology. By Daniel Greenleaf Thompson. 2 vols. (London : Longmans, 1884.)

PSYCHOLOGY, like other sciences, may be regarded as a pure science, or as a set of generalisations capable of application to practice, or as material for a philosophical construction. Mr. Thompson has treated it, for the most part, in the spirit of a scientific inquirer. He does not stop to make applications to practical questions, and although he is not without metaphysical views of his own, it is evident that he is inter-

ested in psychology more for its own sake than for the sake of its bearing on his theory of the universe. There is, therefore, no need to discuss here the questions in dispute between the empirical school to which Mr. Thompson belongs and its various critics. As he has treated psychology so much in the scientific spirit, we may confine ourselves to indicating the kind of work he has done in his own special line.

Some have denied that psychology is a science, on the ground that it does not make progress ; but it is only necessary to compare Locke's "Essay" with any modern work in which the treatment is not altogether inadequate, in order to see that progress has been made both in accuracy of description and in refinement of analysis of psychological facts. The admiration that must be felt for what Locke was able to do only makes the comparison more conclusive so far as the establishment of the scientific character of psychology is concerned. In criticising any new book, then, we ought to ask whether the author has made any advance on his immediate predecessors. We ought, in fact, to apply to the particular author we are criticising the test of progress to which psychology as a whole may be submitted. Mr. Thompson's book will emerge successfully from an examination such as that which is here suggested. In dealing with many special questions he goes beyond the later English psychologists just as they themselves have gone beyond Locke.

A student might very well begin with the sixth part of Mr. Thompson's book, entitled "The General Development of States of Consciousness," in order to get at the author's more important results, and then read the parts that come before it to understand more fully his general view of his subject, and the parts that come after it for new details. In this division of his work, the author brings out very clearly the difference between "presentative" and "representative" states of consciousness, and shows the influence of this difference in the spheres of feeling and of will, as well as of cognition. Emotional states are classified according to their relation to the environment, which may take the form of "pleasurable interest in external objects" or of "aversion to external objects." The chapter on "volitional development" (the first of the second volume) deserves the special attention of the psychological student. Mr. Thompson's introduction into the view he gives of the external world in its relation to mind (in Part III.), of a sort of Cartesian conception of "matter" as including "space," must be at least alluded to as likely to be found interesting both by physicists and metaphysicians. Although philosophy and science are now too much specialised for an idea of this kind to have any direct influence on research, yet all discussion between philosophers and men of science of the more general terminology of the sciences, and especially of physics, must have some effect in compelling clear definition of terms on the part of physicists and at the same time in keeping philosophic thought in contact with its basis of scientific law.

Mr. Thompson might perhaps have given a better account of the introspective method in psychology if he had had fuller possession of the idea of mind as something common to all individuals ; if he had been able to show more clearly that it is not simply the individual